OPTIMIZATION

المدرسة المحمدية للمهندسين
Ecole Mohammadia d'Ingénieurs

A Life-Cycle Optimization Model Using Semi-Markov Process for Highway Bridge Maintenance

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1. Justification for the choice :

Civil engineering is a vast discipline, bridges construction is not an exception, and I have always been fascinated on how bridges are getting built, lifted up in the sky and linked in between. The texture and the architect of them catch my attention, even the feeling of traversing them is an adventure itself.

Bridges are estimated to last between 25 to 80 years depending on location and material. However, bridges may age 100 years with proper maintenance and rehabilitation.

This project presents a life-cycle optimization model using a semi-Markov process and demonstrates how the proposed method can assist highway agencies to make more quantitative and explicit decisions for bridge maintenance applied in the State of Texas.

2. Hightlights:

2.1 Keywords:

Bridge maintenance ; Life-cycle optimization ; Semi-markov process ;

Risks ; Uncertainty ; NBI Database (National Bridge Inventory)

2.2 Annotations :

The following symbols are used in this paper:

$X(t)$ **: Markov process state for any time** t_n

$P^{t,\,t+1}$ **: Markov transition probability matrix**

$A(t)$ **: Probability Mass Function (PMF) at any time** t

$F_i(t)$**: Cumulative Density Function (CDF) of waiting time at any time** t

$S_i(t)$ **: Survival Function (CDF) of waiting time at any time** t

$f_i(t)$ **: Probability Density Function of waiting time at any time** t

R_e **: Repair Effect Matrix**

β_i, λ_i **: Parameters of Weibull Distributions**

3. Introduction of the Problem :

Highway bridges are vital links in the U.S. transportation system, but a large portion of them are deteriorating constantly due to various risks, such as:

Defective design of bridge;

Poor quality of construction materials;

Poor quality control during construction;

Effect of water on the parts in contact with water;

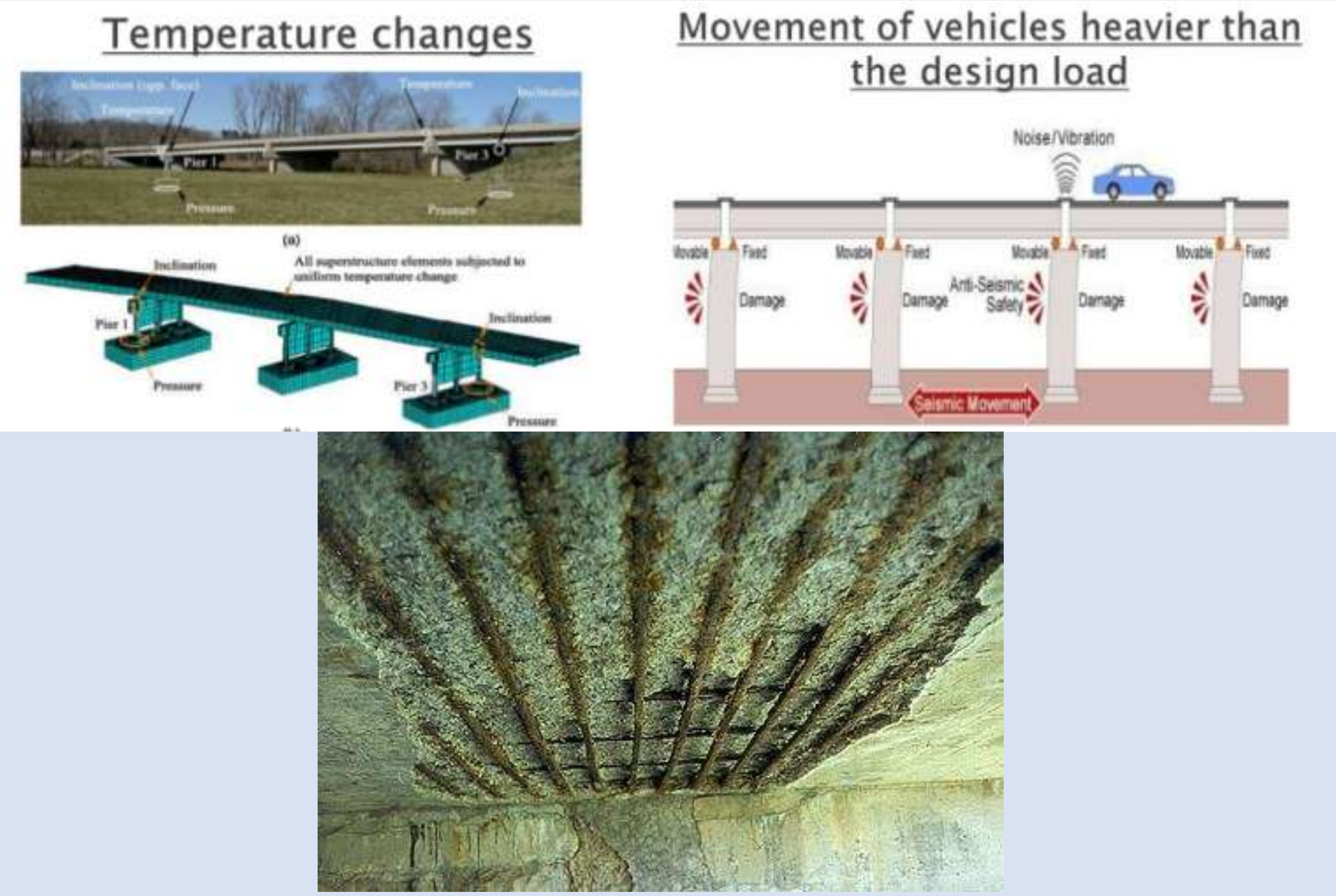
Resistance with weathering effect;

Temperature change;

High traffic volumes;

Miscellaneous factor like flood, earthquake.....etc;

Negligence of minor effects


Temperature changes

Movement of vehicles heavier than the design load

Fig 1 : Common bridge deterioration causes

Problematic :

Repairing these critical pieces of infrastructure would require a significant investment, but most highway agencies lack the necessary funds to address the backlog of bridge repairs and therefore need effective methodologies for allocating limited resources efficiently and cost-effectively.

4. *Examination of the method :*

To overcome these limitations, a life-cycle optimization model using a semi-Markov process is proposed in this study to make the decision-making process of bridge management more quantitative and explicit for highway agencies. Various factors are considered in this model, and the bridge deterioration is assumed as a semi-Markov process, which means that the durations in each condition state, also called state waiting times, are defined as random variables with probability distributions (e.g., the well-known Weibull distribution). The timing and costs of interventions (maintenance activities, rehabilitations, and repairs) in a bridge's life-cycle are determined by the proposed model, and the most efficient and cost-effective procedure can be selected from several alternative maintenance strategies.

Compared with the existing models, the proposed model has at least three unique features: the impacts of various factors, including traffic volumes (or Average Daily Traffic (ADT)), bridge types, construction materials, etc., are considered thoroughly; a semi-Markov chain that relaxes the homogeneous limitation of traditional Markov transition matrices is adopted herein; and a life-cycle cost analysis that considers different repair strategies along the planning horizon is integrated with the model. Through this model, highway agencies have great potential for optimizing their decisions on structuring the bridge life-cycle maintenance plan, leading to cost savings and more efficient sustainability of their transportation infrastructure.

4.1 The basics:

In a discrete-time Markov process as a stochastic process with states $X(t)$, for any n time points $t_1, t_2, \ldots, t_n$, the conditional distribution of $X(t_n)$ for given values of $\{X(t_1), \ldots, X(t_{n-1})\}$ depends only on $X(t_{n-1})$, which is the most recent known value. This can be stated as:

$$\begin{aligned} &\mathrm{P}\left[X(t_n) \leq x_n \middle| X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_n) = x_n\right] \\ &= \mathrm{P}\left[X(t_n) \leq x_n \middle| X(t_1) = x_1\right]. \end{aligned} \tag{1}$$

When the Markov process goes from state X_i to state X_j, it is said to transition from state *i* to state *j*. A Markov transition probability matrix denotes the transition probability from state *i* to state *j* during time *t*, as shown below:

$$P^{t,t+1} = \begin{bmatrix} p_{11}^{t,t+1} & p_{12}^{t,t+1} & \dots & p_{1n}^{t,t+1} \\ p_{21}^{t,t+1} & p_{22}^{t,t+1} & \dots & p_{2n}^{t,t+1} \\ \dots & \dots & \dots & \dots \\ p_{n1}^{t,t+1} & \dots & \dots & p_{nn}^{t,t+1} \end{bmatrix}; p_{ij}^{t,t+1} \geq 0\,(i,j = 1,2,\dots,n);$$
$$\textstyle\sum_{j=1}^{n} p_{ij}^{t,t+1} = 1\,(i = 1,2,\dots,n). \tag{2}$$

In an *n*-state space discrete Markov process, the state of the process at any time *t* is typically stochastic and is defined by a Probability Mass Function (PMF) that is denoted by an *n*-dimensional vector *A(t)*.

$$A(t) = \{a_1^t, a_2^t, \dots a_n^t\}\left(\textstyle\sum_{i=1}^{n} a_i^t = 1\right). \tag{3}$$

Member a_i^t denotes the probability that the process is in state *i* at time *t*. The PMF of the process at time ($t + k$) is obtained:

$$A(t+k) = A(t)\, P^{t,t+1} P^{t+1,t+2} \dots P^{t+k-1,t+k}. \tag{4}$$

For a stationary Markov chain, the above equation can be reduced to:

$$A(t+k) = A(t)\, P^{k}. \tag{5}$$

In which the Markov chain is stationary in time (or homogeneous) and the Markov transition probability matrices are constant.

Usually, under the condition of no interventions (renewals or rehabilitations), the process of bridge degradation is unidirectional, i.e., if state *1* denotes good as new and state *n* denotes failure, then the process can move only from state *i* to state *j*. Further, it is often assumed that a bridge can deteriorate only one state at a time, that is, the bridge will deteriorate from state 1 to state 2, then to state 3, and so on to failure. The process thus cannot jump from state 1 to state 3 without passing through state 2. If a bridge is expected to deteriorate very fast, one can shorten transition periods to the extent that realistically only single state deterioration is possible in one period. The time period is assumed to be one or two years, since highway agencies normally inspect bridges every other year to update the NBI dataset. This leads to a relatively simple Markov transition probability matrix:

$$P^{t,t+1} = \begin{bmatrix} p_{11}^{t,t+1} & p_{12}^{t,t+1} & 0 & \dots & 0 \\ 0 & p_{22}^{t,t+1} & p_{23}^{t,t+1} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & p_{n-1,n-1}^{t,t+1} & p_{n-1,n}^{t,t+1} \\ 0 & 0 & \dots & 0 & p_{n,n}^{t,t+1} \end{bmatrix}. \qquad (6)$$

4.2 Semi-Markov Process :

In the real practice, a bridge deterioration process in which sojourn (or waiting) times in any given state are time-dependent distributed random variables cannot be captured by a discrete time Markov chain. In this regard, a semi-Markov chain with various transition probability matrices can relax this limitation and is more suitable to capture the bridge's real degradation process. A semi-Markov process is a class of stochastic processes that moves from one state to another with the successive states visited forming a Markov chain. The process stays in a particular state for a random length of time, and its distribution depends on the state and on the next state to be visited.

In a semi-Markov process, $T_1, T_2, \dots, T_{n-1}$ are random variables denoting the sojourn times in states $\{1, 2 \dots n-1\}$, respectively. Their corresponding Probability Density Functions (PDFs), Cumulative Density Functions (CDFs), and Survival Functions (SFs) are thus denoted $f_i(t)$, $F_i(t)$, and $S_i(t)$. $T_{i\text{-}k}$ is the time it will take the process to go from state i to state k. In addition, $f_{i\text{-}k}(T_{i\text{-}k})$, $F_{i\text{-}k}(T_{i\text{-}k})$, $S_{i\text{-}k}(T_{i\text{-}k})$ are the PDF, CDF, and SF of $T_{i\text{-}k}$, respectively. If the deterioration process is in state 1 at time t, the conditional probability that it will transit to the next state in the next time step Δt is given by:

$$\Pr\left[X(t+1) = 2 \middle| X(t) = 1\right] = p_{1,2}(t) = \frac{f_1(t)\Delta t}{S_1(t)}, \qquad (7)$$

Where $t = 0$ is the time when the process enters into state 1 (i.e., new asset – in most cases). It should be noted that Δt is assumed to be small enough to exclude a two-state deterioration. In this case study, Δt is assumed to be one unit (year) and is thus omitted. Similarly, if the process is in state 2 at time t, the conditional probability that it will transit to the next state in the next time step Δt is given by:

$$\Pr\left[X(t+1) = 3 \middle| X(t) = 2\right] = p_{2,3}(t) = \frac{f_{1\to 2}(t)}{S_{1\to 2}(t) - S_1(t)}. \qquad (8)$$

Noted that in Eq (8), the PDF $f_{1\rightarrow 2}(t)$ pertains to $T_{1\rightarrow 2}$, which is the random variable denoting the sum of waiting times in states 1 and 2. Further, the denominator expresses the simultaneous condition that $T_{1\rightarrow 2} < t$ and $T_1 < t$, which is equivalent to the condition $X(t) = 2$. In a general form, if the deterioration process is in state *i* at time *t,* the conditional probability that it will transit to the next state $i + 1$ in the next time step Δt (usually one unit (year)) is given by:

$$\Pr\left[X(t+\Delta t)=i+1|\mathrm{X}(\mathrm{t})=\mathrm{i}\right]=p_{i,i+1}\left(t\right)=\frac{f_{1\rightarrow i}(t)\Delta t}{S_{1\rightarrow i}(t)-S_{1\rightarrow (i-1)}(t)} \tag{9}$$

$$or\ \Pr[\mathrm{X}(\mathrm{t}+1)=\mathrm{i}+1|\mathrm{X}(\mathrm{t})=\mathrm{i}]=p_{\mathrm{i},i+1}\left(t\right)=\frac{f_{1\rightarrow i}(t)}{S_{1\rightarrow i}(t)-S_{1\rightarrow (i-1)}(t)}$$

$$(i=1,2,\ldots,n-1)\,.$$

Eq 9, provides all the transition probabilities $p_{i,i+1}(t)$ to generate the transition probability matrix for the semi-Markov process. It is noted that these transition probabilities are assumed to be time-dependent, and thus the process is non-stationary (or non-homogeneous). Once the transition probability matrix is established, the deterioration process can be modeled to obtain the PMF of the process at any time *t.*

The waiting time T_i of the process in any state *i* is arbitrarily modeled as a random variable with a two-parameter Weibull probability distribution. Since the PDF $(f_{i\text{-}k}(T_{i\text{-}k}))$,CDF$(F_{i\text{-}k}(T_{i\text{-}k}))$,and SF$(S_{i\text{-}k}(T_{i\text{-}k}))$ of the sum of waiting times cannot be calculated analytically, a Monte-Carlo simulation needs to be implemented to numerically calculate these functions for sums of Weibull-distributed random variables $T_{i\text{-}k}$ according to Eq10;

$$F_i\left(t\right)=\Pr\left[T_i\leq t\right]=1-e^{-(\lambda_i t)^{\beta_i}} \tag{10}$$

$$S_i\left(t\right)=1-F_i\left(t\right)=e^{-(\lambda_i t)^{\beta_i}} \tag{11}$$

$$f_i\left(t\right)=\frac{\delta F_i(t)}{\delta t}=\lambda_i\beta_i\left(\lambda_i t\right)^{\beta_i-1}e^{-(\lambda_i t)^{\beta_i}}. \tag{12}$$

The parameters for the deterioration model are based on historical observations and condition assessments of bridges since there are sufficient historical records in the NBI database. The parameters are then derived by Eq13 ;

$$\begin{cases} S_i\left(u\right)=e^{-(\lambda_i u)^{\beta_i}} \\ S_i\left(v\right)=e^{-(\lambda_i v)^{\beta_i}} \end{cases}\rightarrow\begin{cases} \ln\left[S_i\left(u\right)\right]=-(\lambda_i u)^{\beta_i} \\ \ln\left[S_i\left(v\right)\right]=-(\lambda_i v)^{\beta_i} \end{cases}\rightarrow\ln\left\{\frac{\ln[S_i(u)]}{\ln[S_i(v)]}\right\}=\beta_i\ln\left(\frac{u}{v}\right), \tag{13}$$

where

$$\beta_i = \frac{\ln(\ln[(S_i(u))] - \ln[(S_i(v))])}{\ln(u) - \ln(v)}; \lambda_i = \frac{1}{u}(-\ln[S_i(u)])^{\frac{1}{\beta_i}}. \tag{14}$$

In the above equations, if the probability of $S_i(u)$ and $S_i(v)$ of being in state i at the uth and vth years are known, parameters β_i and λ_i can be derived. Once parameters β_i and λ_i are known, the transition probability matrix can be established. The determination of these parameters will be presented in detail in the case study given later.

4.3 The definition of semi-Markov state :

According to the FHWA bridge condition rating , five semi-Markov states for the degradation of bridges can be defined as follows: "State 1" is good as new (above 90% sufficiency), "State 2"(80–90% sufficiency), "State 3" (50–80% sufficiency), "State 4" (20–50% sufficiency), and "State 5" (less than 20% sufficiency). Also, repair strategies are correspondingly determined to be "Do-Nothing", "Preventative Maintenance, i.e., "Repainting", "Minor Repair", "Major Repair", and "Replacement", for these five states, respectively.

4.4 Prediction of life cycle cost :

The Markov chain can be used to predict future costs of bridge rehabilitation activities within the life-cycle term. The expected cost of failure at age t is the product of the cost of failure and the probability of failure at age t

$$E[C^F(t)] = C^F a_n^t. \tag{15}$$

It is assumed that the rehabilitation or repair can be modeled by forcing the current state of the bridge back to its previous states. This operation can be done systematically with the following Markov transition matrix $[R_e]$:

$$R_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0.1 & 0 & 0 \\ 0.8 & 0 & 0 & 0.2 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix}.$$

It should be noted that the repair effects are subject to the restrictions of the bridge's current conditions (i.e., the bridges in State 2 should have a greater likelihood to return to State 1 than those in State 3) and repair strategies (i.e., a major repair activity should have a greater likelihood to restore bridge conditions than a minor repair activity). Therefore, it is reasonable to assume that there are 90%, 80%, and 50% probabilities to return to State 1 for bridges in States 3, 4, and 5, respectively. Based on these probabilities, the expected costs can be calculated by the following equation considering the effects of repair activities:

$$E\left[C^F(t)\right] = \sum C^F A_0 T^t R_e (1+i)^{-t}, \tag{16}$$

Where $E[C^F(t)]$ denotes the expected cost of failure at age t; R_e denotes the repair effect matrix; and i denotes the discount rate. The life-cycle costs of bridge maintenance based on the proposed model will be discussed in detail in the case study given later.

4.5 Algorithm :

- The algorithm of the proposed model is summarized by the following step-by-step procedures and also illustrated in a flowchart in ***Fig2*** for better understanding:
- .
- Step 1: Define the semi-Markov states in the bridge deterioration process according to the FHWA bridge condition rating.
- .
- Step 2: Determine the parameters of the Weibull distributions of state waiting times by using the NBI historical data or Monte-Carlo Simulation if historical data is not readily available.
- .
- Step 3: Establish the semi-Markov transition probability matrix based on the known Weibull-distributions of state waiting times.
- .
- Step 4: Make bridge maintenance strategies based on existing bridge repair techniques and sound engineering judgment.
- .
- Step 5: For each strategy, determine the PMF at the year when repair activity occurs by using the transition probability matrices.
- .
- Step 6: Calculate the repair costs according to the PMF of the year when repair activity occurs.
- .
- Step 7: Update the PMF of that year by using the repair matrix to reflect the repair effects.
- .
- Step 8: Repeat Step 5–7 if there is another repair activity. Otherwise, sum the costs of all repair activities to obtain the life-cycle costs of each strategy.
- .
- Step 9: Evaluate the repair effects of maintenance strategies and then determine the most optimal strategy based on cost and structural effect. Within the constraint of limited funding, the most optimal strategy should produce the highest structural performance in the whole lifespan of bridge while generating the minimum life-cycle costs.

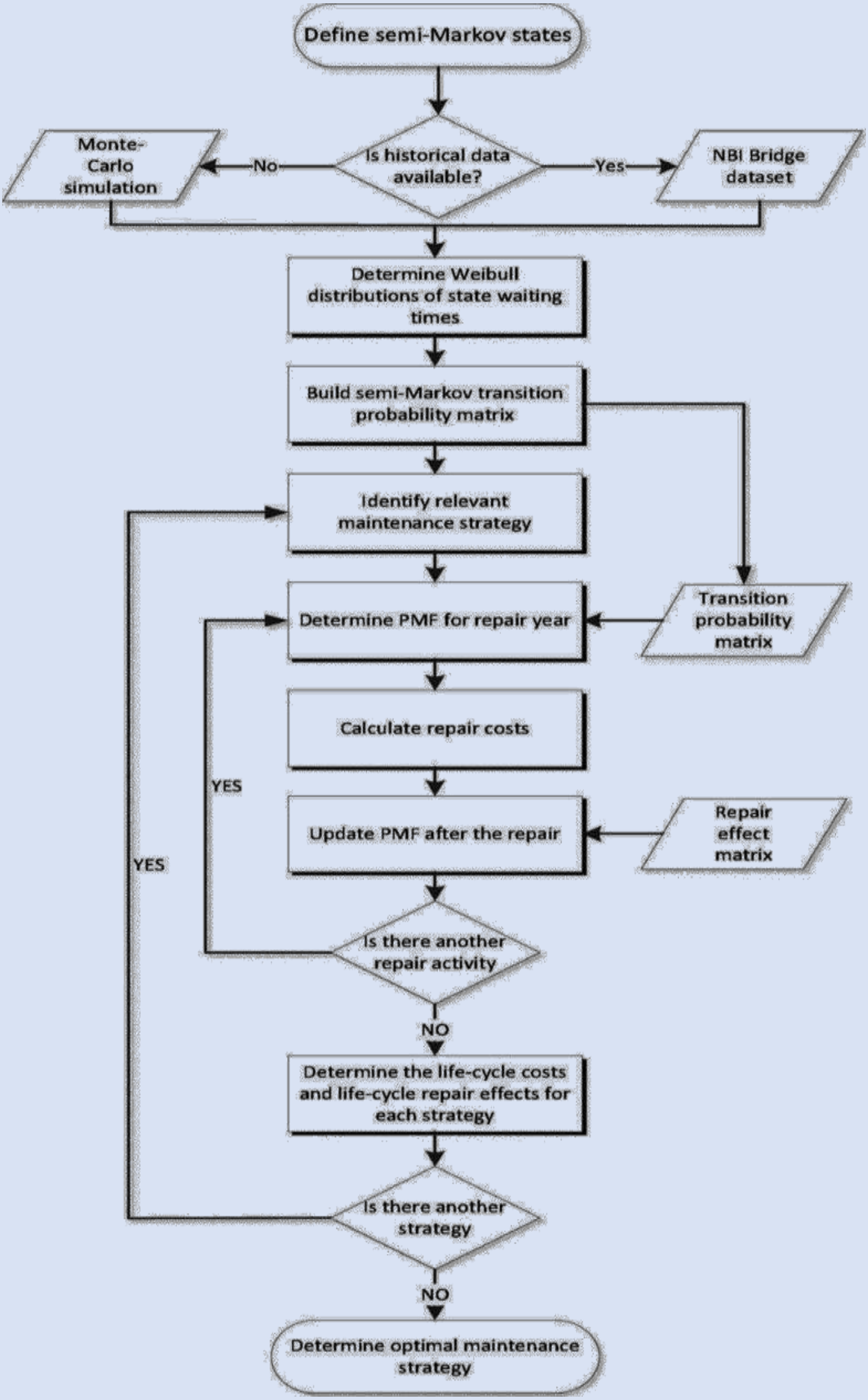

Define semi-Markov states
Is historical data available?
No
Yes
Monte-Carlo simulation
NBI Bridge dataset
Determine Weibull distributions of state waiting times
Build semi-Markov transition probability matrix
Identify relevant maintenance strategy
Determine PMF for repair year
Transition probability matrix
Calculate repair costs
Update PMF after the repair
Repair effect matrix
YES
YES
Is there another repair activity
NO
Determine the life-cycle costs and life-cycle repair effects for each strategy
Is there another strategy
NO
Determine optimal maintenance strategy

Fig 2 : Common bridge deterioration causes

5. Case study / Application of the algorithm in Texas state :

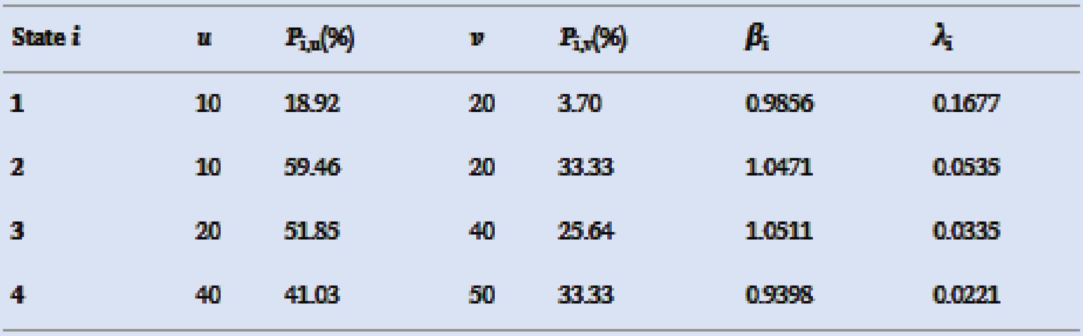
5.1 Purpose :

This case study focuses on the performance and maintenance of steel bridges carried by local routes in Texas, and its purpose is to illustrate: 1) how to apply the proposed model to formulate the semi-Markov chain and then to predict the bridge deterioration process based on the model; 2) how to calculate the life-cycle costs of different maintenance strategies based on the semi-Markov probability matrices; 3) how to evaluate the effects of rehabilitation activities and the post-repair behaviors of bridges; and 4) how to optimize the life-cycle bridge maintenance based on efficacy(or post-repair performance) and funding. In this example, only steel bridges without reconstruction are retrieved from the 2012 NBI dataset for Texas, which makes the deterioration of bridges in the service life a unidirectional decreasing curve. A discount rate of 3.5% is assumed, and the analysis period is 75 years, which is long enough to incorporate the cost of at least one rehabilitation activity for all alternatives.

In this case study, five maintenance strategies have been proposed. However, for illustrative purposes, only Strategy A is presented in detail. Other strategies can be analyzed in the same way, so only the results are presented. The analysis procedure includes:

- Step 1: Define five semi-Markov states as discussed previously: "State 1" is good as new (above 90% sufficiency), "State 2" (80–90% sufficiency), "State 3" (50–80% sufficiency), "State 4" (20–50% sufficiency), and "State 5" (less than 20% sufficiency).

- Step 2: Determine the parameters of the Weibull distributions of state waiting times: the probabilities of any two different years for each state can be determined from the 2012 NBI dataset for Texas, and then the parameters of the Weibull-distributed holding times are summarized in Table **1;**



State i	u	$P_{i,u}$(%)	v	$P_{i,v}$(%)	β_i	λ_i
1	10	18.92	20	3.70	0.9856	0.1677
2	10	59.46	20	33.33	1.0471	0.0535
3	20	51.85	40	25.64	1.0511	0.0335
4	40	41.03	50	33.33	0.9398	0.0221

Table1; The parameters of weibull-distributed random variables *Ti-k*.

Next, the waiting times T_i in every state *i* and the sums of waiting times in the various states, $T_{i\text{-}k}$, are then calculated by these parameters and, (9) and (10), as shown in ***Fig3*** and ***4*** for the PDFs and SDFs, respectively.

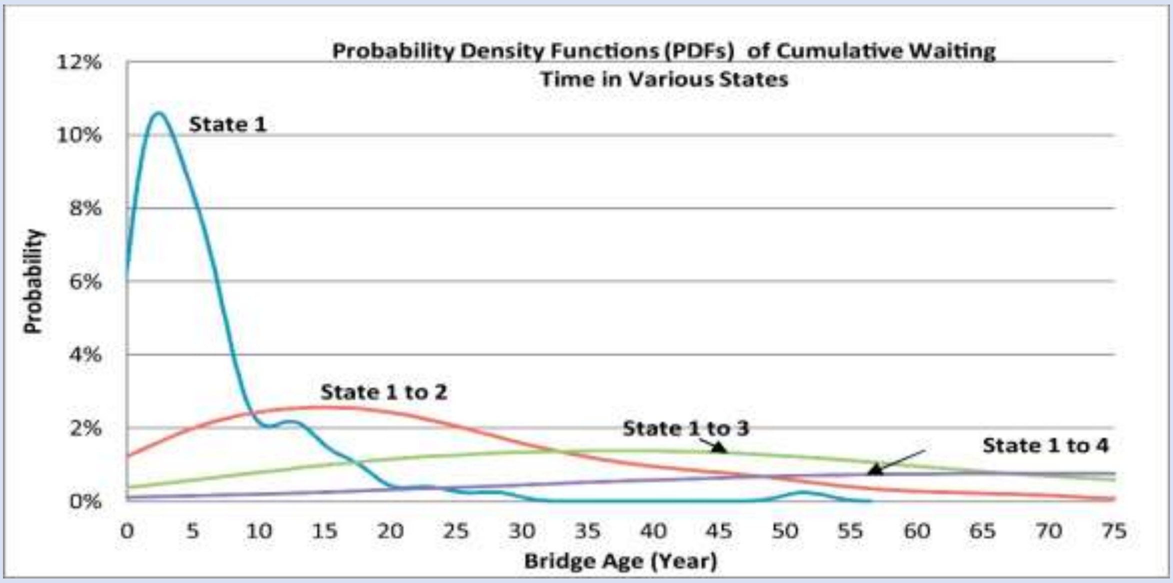


Fig. 3. Cumulative waiting times in various states.

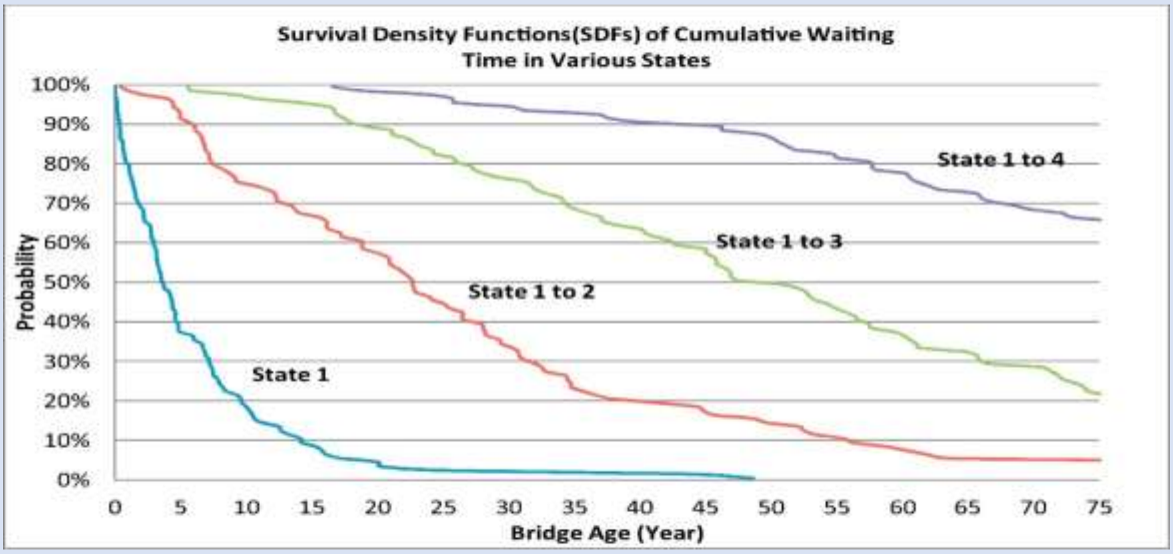


Fig. 4. Survival distribution functions of cumulative waiting times in various states.

- Step 3: Once the Weibull distributions of state waiting times are determined, the age-dependent transition probability matrices can be calculated by Eq (9) from Year 1 to Year 75. A typical transition probability matrix, i.e., the transition matrix between Year 2 and Year 3, is shown below:

$$p^{2,3} = \begin{bmatrix} 0.9991 & 0.0009 & 0 & 0 & 0 \\ 0 & 0.9994 & 0.0006 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

If these transition probabilities are determined, the deterioration process can be formulated as:

$$A(t) = A(t_0)\prod_{k=0}^{t-1} P_{ij}^{t_0+k,t_0+k+1}, \tag{17}$$

Where *A(t)* is the PMF of the bridge at any time *t* in the life span. For example, the PMF of Year 5 is:

$$\begin{aligned} A(5) &= A(1)\prod_{k=0}^{4} P_{ij}^{k,k+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.9998 & 0.0002 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\ & \begin{bmatrix} 0.9996 & 0.0004 & 0 & 0 & 0 \\ 0 & 0.9996 & 0.0004 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} . \\ & \begin{bmatrix} 0.9991 & 0.0009 & 0 & 0 & 0 \\ 0 & 0.9994 & 0.0006 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.9983 & 0.0017 & 0 & 0 & 0 \\ 0 & 0.9993 & 0.0007 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.9880 & 0.0120 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0.9938 & 0.0062 & 0 & 0 & 0 \\ 0 & 0.9971 & 0.0029 & 0 & 0 \\ 0 & 0 & 0.9988 & 0.0012 & 0 \\ 0 & 0 & 0 & 0.8981 & 0.1019 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

- Step 4: Make bridge maintenance strategies based on available techniques and existing experiences. In this case study, five bridge maintenance strategies (Strategies A, B, C, D and E) are considered, and the detailed repair plans are shown in ***Fig. 5***. For the illustrative purpose, only the minor repair cost is included, and other costs are assumed to be zero: C_A = [\$0, \$0, \$29,000, \$0, \$0]^T, C_B = [\$0, \$0, \$68,000, \$0, \$0]^T, C_C = [\$0, \$0, \$32,000, \$0, \$0]^T, C_D = [\$0, \$0, \$44,000, \$0, \$0]^T, C_E = [\$0, \$0, \$144,000, \$0, \$0]^T, for Strategies A, B, C, D and E, respectively. Noted that the relevant information and cost data are obtained from a real bridge repair project and more details can be referred to the author's work.

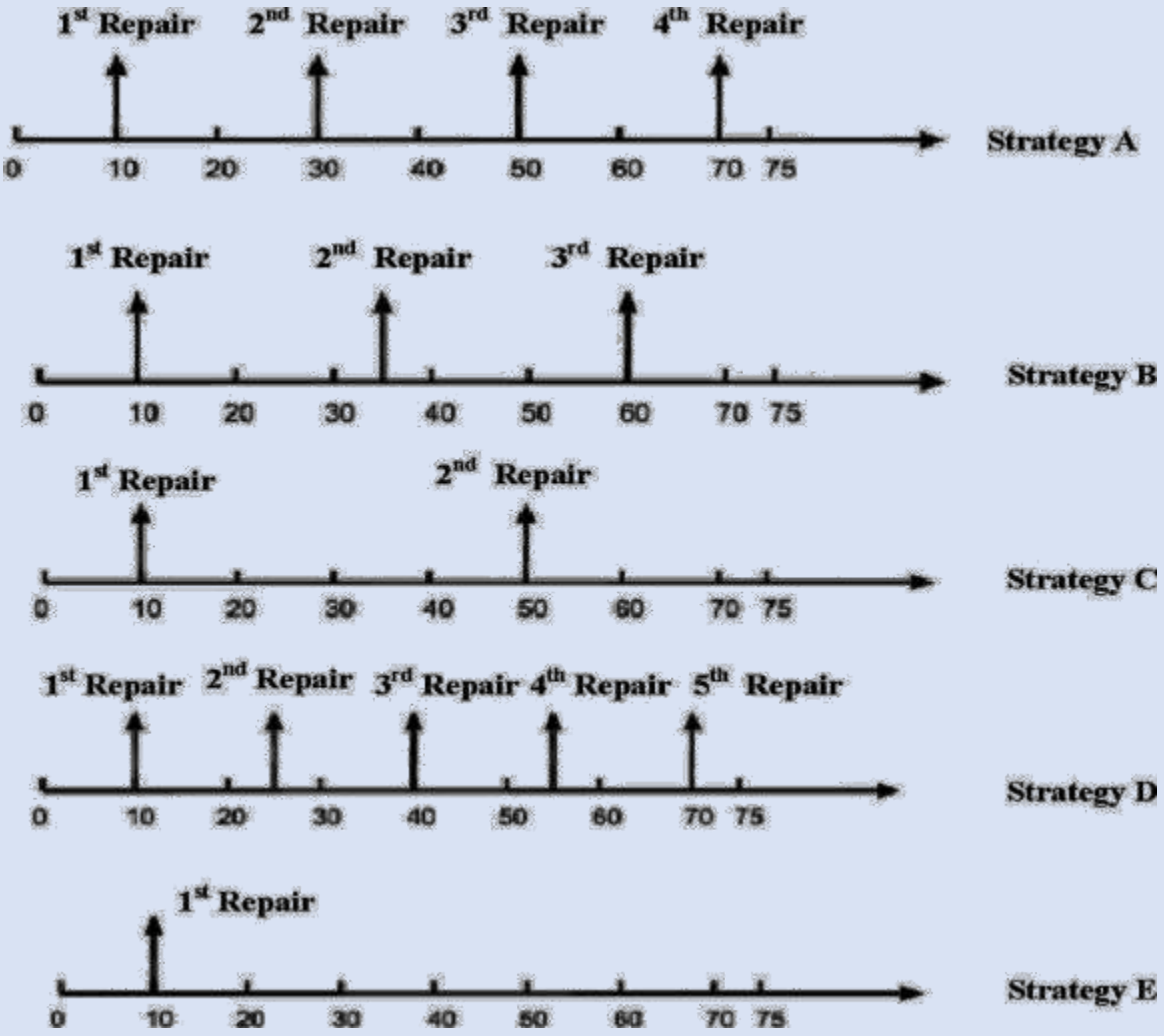


Fig. 5. Repair plans of strategies A, B, C, D, and E.

- Step 5: For each strategy, determine the PMF at the year when the repair occurs by using transition probability matrices. Using Strategy A as the example, the first repair activity occurs in Year 10 and the PMF of five states in Year 10 can be found in Table 1;

- Step 6: Assuming the discount rate is 3.5%, calculate the repair cost by the PMF of the year when the repair occurs. Based on Strategy A, the expected cost in Year 10 can be calculated as:

$$
\begin{aligned}
E[C^{10}] &= A(10)^{*}C^{A^{*}}(1+3.5\%)^{-10}\\
&= \begin{bmatrix} 0.9192 & 0.0798 & 0.0010 & 0 & 0 \\ 0 & 0.9684 & 0.0315 & 0.0001 & 0 \\ 0 & 0 & 0.9917 & 0.0079 & 0.0004 \\ 0 & 0 & 0 & 0.7975 & 0.2025 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 29,000 \\ 0 \\ 0 \end{bmatrix} (1+3.5\%)^{-10} = \$3,561.
\end{aligned}
$$

- Step 7: Update the PMF of that year by using the repair matrix to reflect the repair effect. After the repair of Year 10, the new PMF of states will be improved as:

$$A(10)^{New} = A(10)^{*} R = \begin{bmatrix} 0.9192 & 0.0798 & 0.0010 & 0 & 0 \\ 0 & 0.9684 & 0.0315 & 0.0001 & 0 \\ 0 & 0 & 0.9917 & 0.0079 & 0.0004 \\ 0 & 0 & 0 & 0.7975 & 0.2025 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0.9 & 0 & 0.1 & 0 & 0 \\ 0.8 & 0 & 0 & 0.2 & 0 \\ 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix}.$$

$$= \begin{bmatrix} 0.9999 & 0 & 0.0001 & 0 & 0 \\ 0.9968 & 0 & 0.0032 & 0 & 0 \\ 0.8991 & 0 & 0.0992 & 0.0016 & 0.0002 \\ 0.7392 & 0 & 0 & 0.1595 & 0.1013 \\ 0.5000 & 0 & 0 & 0 & 0.5000 \end{bmatrix}$$

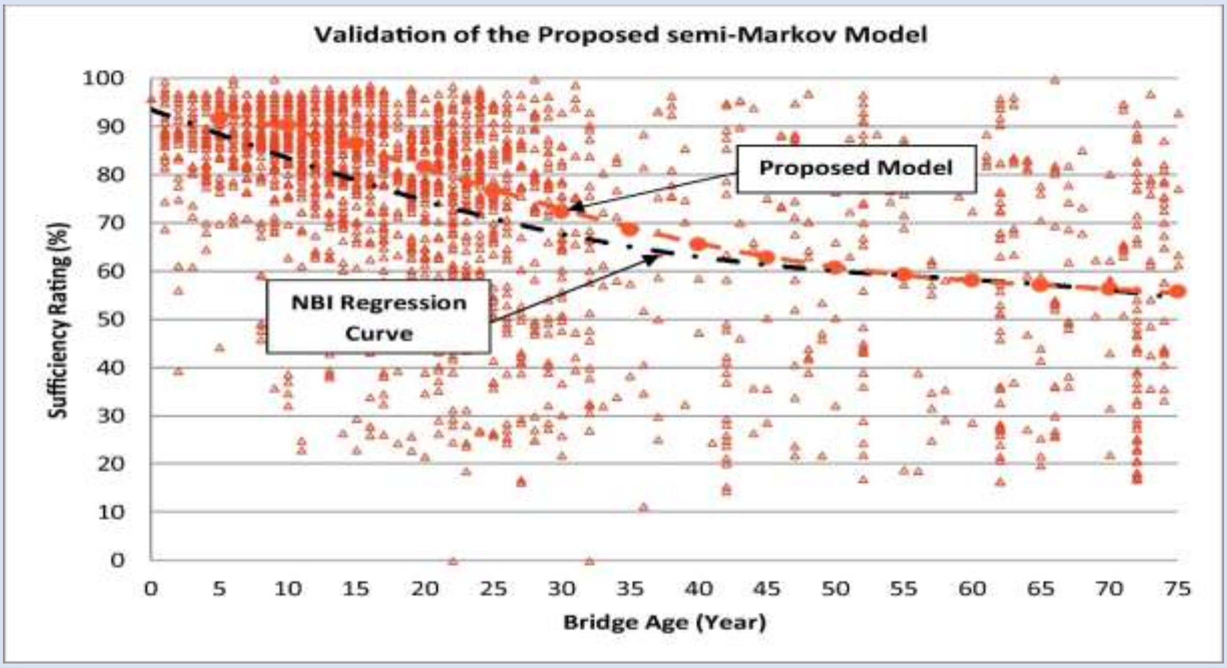


Fig. 6. Validation of the proposed deterioration model and the 2012 NBI historical records for Texas.

- Step 8: Sum the costs of all repair activities to obtain the life-cycle costs for each repair strategy (Strategies A, B, C, D, and E), as shown in ***Fig7***. The order of life-cycle costs from the highest to the lowest is: **Strategy E > B > D > C > A**.

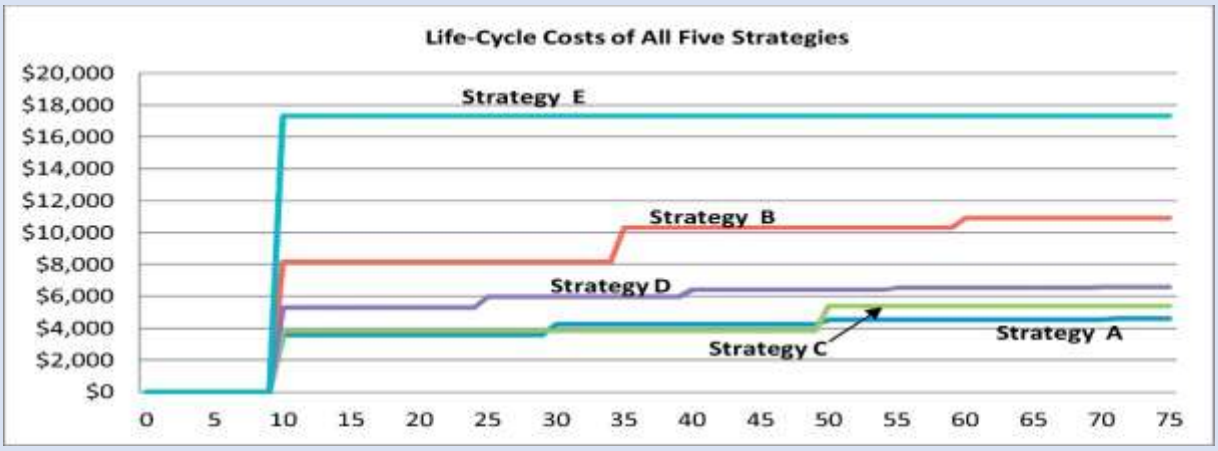


Fig. 7. Life-cycle costs for Al five strategies in the 75-year bridge service life.

- Step 9: Evaluate the repair effects of all maintenance strategies and then determine the most optimal strategy considering both the repair effect (and post-repair performance) and funding (the one with the lowest costs). Since the states in the proposed model are defined by a Sufficiency Rating (SR) that is mainly comprised of structural adequacy, it is reasonable to assume that the bridge, while in different states, has different load capacities, i.e., the bridge in State 1 has 100% of the original design capacity, while the bridge in State 5 only supports 20% of the design load capacity. The load capacities for all five states can be written in the form of a matrix: $R_e = [100\%, 80\%, 60\%, 40\%, 20\%]^T$. If the PMF ($A(t)$) of the bridge at the year when the repair occurs is known, the repair effect can be evaluated by the following equation based on the restored load capacity:

$$P^{new} = A\left(t\right) R_e \tag{18}$$

In which P_{new} is the restored load capacity after the repair, R_e is the defined repair effect matrix in terms of load capacity, and $A(t)$ is the PMF of the year when the repair is applied.

Again using Strategy A as an example, the load capacity of the bridge in Year 1 can be calculated as:

$$P = sum\left(A\left(1\right)\left[R_e\right]/300\%\right) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100\% \\ 80\% \\ 60\% \\ 40\% \\ 20\% \end{bmatrix} = 100\%.$$

It should be noted that P is normalized to 100% of the design capacity by the division of 300%.
At Year 10 before the repair, the load capacity is decreased by the deterioration:

$$P = sum\left(A\left(10\right)\left[R_e\right]/300\%\right)$$
$$= \begin{bmatrix} 0.9192 & 0.0798 & 0.0010 & 0 & 0 \\ 0 & 0.9684 & 0.0315 & 0.0001 & 0 \\ 0 & 0 & 0.9917 & 0.0079 & 0.0004 \\ 0 & 0 & 0 & 0.7975 & 0.2025 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 100\% \\ 80\% \\ 60\% \\ 40\% \\ 20\% \end{bmatrix} = 97.84\%.$$

When the first repair is applied, the PMF of the bridge condition is improved, and thus the load capacity is increased correspondingly:

$$P^{new} = sum\left(A(10)^{new}\left[R_e\right]/300\%\right)$$
$$= \begin{bmatrix} 0.9999 & 0 & 0.0001 & 0 & 0 \\ 0.9968 & 0 & 0.0032 & 0 & 0 \\ 0.8991 & 0 & 0.0992 & 0.0016 & 0.0002 \\ 0.7392 & 0 & 0 & 0.1595 & 0.1013 \\ 0.5000 & 0 & 0 & 0 & 0.5000 \end{bmatrix} \begin{bmatrix} 100\% \\ 80\% \\ 60\% \\ 40\% \\ 20\% \end{bmatrix} = 146.04\%.$$

For other repair activities aside from Strategy A, the bridge load capacities after the repair can be calculated similarly. Then, the repair effects for all five strategies are calculated and presented in ***Fig8;***

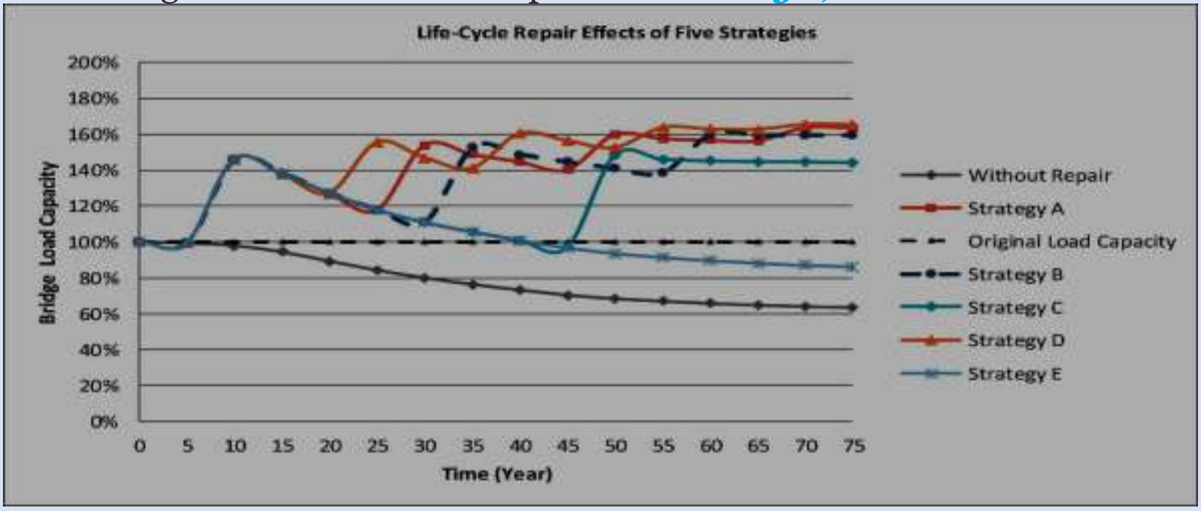


Fig. 8. Life-cycle repair effects for five strategies in the 75-year bridge service life.

6. Conclusion :

According to ***Fig8***, the bridge load capacity decreases continuously during the bridge deterioration process without any repair activities. If the repair is applied, strategies A, B, C and D can restore the bridge load capacity to its original design capacity or even increase it to a higher level during the whole life-cycle. The figure indicates that Strategy A improves the bridge to its highest load capacity at the end of its service life. However, Strategy E cannot restore the bridge to its original capacity during its whole life-cycle. The figure also shows that the bridge maintained by Strategy E has a lower load capacity than it did originally around 45 years after its construction. Considering the life-cycle costs of each strategy, Strategy A is still the most optimal.

7. Application in Morocco :

In Morocco there is currently some works concerning the Laayoune bridge, a part of the Tiznit-Dakhla highway that it is under construction, it will be 1650 meters in length, costing a total of (MAD 1, 1 billion). The same bridge is implementing some optimization technique at its very first states one of them is what it was discussed early on. The Data’s are not available yet. However it remains a crucial to anticipate future deterioration that other bridges in morocco are suffering from especially the old ones.

The 1,650 meter long bridge is part of broader efforts to connect Europe with sub-Saharan Africa through a high-quality expressway

8. Enlargement potentials for the optimization method

There are plenty of mathematical approaches to demonstrate the outcome of bridges and their efficiency via differential equations, Markov-chains and other…which are typically described over time. They describe how a system behaves incepting from an initial given condition within a specified time period arriving all the way to a certain state.
This method use Semi-Markov chain, the usual one which is Markov chain is more complicated to determinate the future state because it requires more data's over time which something that we don't acquire yet. In Netherlands for instance the inspections of bridges are carried out periodically and their results are registered in an electronic database. On the basis of visual inspections, bridges are rated on a discrete scale ranging from a perfect condition to a very bad condition (failure). With that it will be less complicated To determine the optimal model parameters, the likelihood function of the data was derived and the maximum likelihood estimator was used. The research presents different approaches for determining the inspector errors and their results via Hidden Markov chain.

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